Junior Maths Mastery Challenge Sample

Paper F

Section A

Questions 1 to 5 carry 3 marks each.

1. Find the value of $\dfrac{1}{\frac{2}{\frac{3}{4}}}$.

[Fractions / Arithmetic]

$2 \div \frac{3}{4} = 2 \times \frac{4}{3}$
$= \frac{8}{3}$
$1 \div \frac{8}{3} = 1 \times \frac{3}{8}$
$= \frac{3}{8}$

(A) $\frac{3}{8}$ (B) $\frac{2}{3}$ (C) $2\frac{2}{3}$

(D) 6 (E) None of the above

2. Helen wants to cut a 5-metre ribbon into shorter pieces of length 0.3 metre or 0.8 metre without any length of ribbon left over. How many ways can she cut the ribbon?

We can list the number of 80-cm pieces to be cut and check if the remaining length of ribbon is divisible by 30.

[Decimals / Make a List]

Number of 80-cm piece	Remaining length of ribbon	Check
1	420 cm	✓
2	340 cm	×
3	260 cm	×
4	180 cm	✓
5	100 cm	×

She can cut the ribbon in 2 ways.

(A) 1 (B) 2 (C) 3

(D) 4 (E) 5

3. Tim and Paul had sports cards in the ratio 3 : 4.
After Paul gave some cards to Tim, the ratio of the number of Paul's cards to that of Tim's cards became 1 : 2. What was the smallest possible number of cards Paul gave Tim?

Before

Tim	:	Paul	Total
3 (×3)		4 (×3)	7 (×3)
9	;	12	21

After

Tim	:	Paul	Total
2 (×7)		1 (×7)	3 (×7)
14	;	7	21

[Ratio / Before-After Concept]
12 – 7 = 5
Paul gave 5 units of cards to Tim.

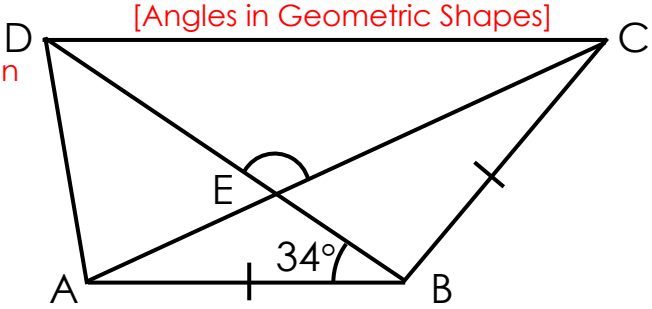
The smallest possible number of cards Paul gave Tim is if 1 unit = 1 card.

The smallest possible number of cards was 5.

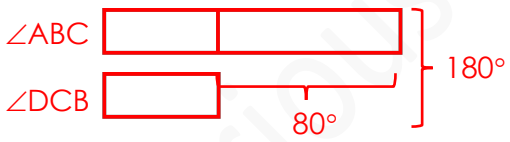
(A) 3 (B) 5 (C) 7
(D) 10 (E) None of the above

4. In the figure, AB is parallel to DC and AB = BC.
The difference in angle size between ∠ABC and ∠DCB is 80°.
Find ∠DEC.

[Angles in Geometric Shapes]



∠ABC + ∠DCB = 180° (The sum of angles between two parallel lines is 180°.)

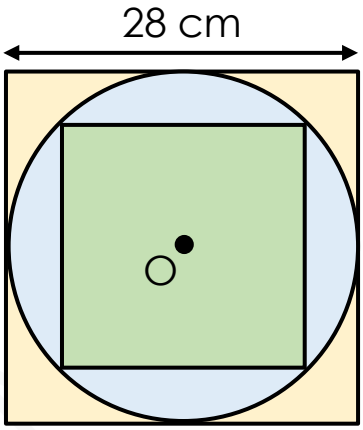


2 units = 180° – 80°
= 100°
1 unit = 100° ÷ 2
= 50°
∠ABC = 50° + 80°
= 130°

∠CAB = (180° – 130°) ÷ 2
= 25°
(The sum of angles in a triangle is 180°. / The base angles of an isosceles triangle are equal.)
∠AEB = 180° – 25° – 34°
= 121°
(The sum of angles in a triangle is 180°.)
∠DEC = ∠AEB
= 121°
(Vertically opposite angles are equal.)

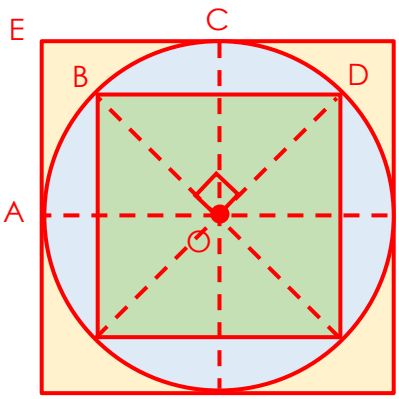
(A) 116° (B) 121° (C) 126°
(D) 131° (E) None of the above

5. The figure below shows two squares and a circle. Point O is the centre of the figure. Find the area of the smaller square.



[Area and Circumference of a Circle / Simplify the Problem]

We can draw some dotted lines to show that the area of the smaller square is half of the area of the larger square.



Observe that the larger square can be divided as shown into 4 identical squares of the same size as Square ECOA.

Observe that OA = OB = OC = OD because they are radii of the circle.

This also shows that the area of Triangle BOD is half of the area of Square ECOA.

Since the smaller square (green) can be divided into 4 identical triangles of the same size as Triangle BOD, the smaller square has an area half of the area of the larger square.

$28 \times 28 = 784$
The area of the larger square is 784 cm^2.

$784 \div 2 = 392$
The area of the smaller square is 392 cm^2.

(A) 196 cm^2 (B) 392 cm^2 (C) 490 cm^2

(D) 588 cm^2 (E) None of the above

Questions 6 to 10 carry 4 marks each.

6. Study the number pattern.

$$\begin{aligned} 1 &= 1 = \frac{1\times 2}{2} \\ 1+2 &= 3 = \frac{2\times 3}{2} \\ 1+2+3 &= 6 = \frac{3\times 4}{2} \\ 1+2+3+4 &= 10 = \frac{4\times 5}{2} \\ 1+2+3+4+5 &= 15 = \frac{5\times 6}{2} \\ &\vdots \end{aligned}$$

Find the largest possible value of n such that $1 + 2 + 3 + 4 + \ldots + n < 200$. [Algebra / Look for Patterns]

We need to find the product of two consecutive numbers that when divided by 2 is smaller than 200.

In other words, the product of the two consecutive numbers is smaller than 400.

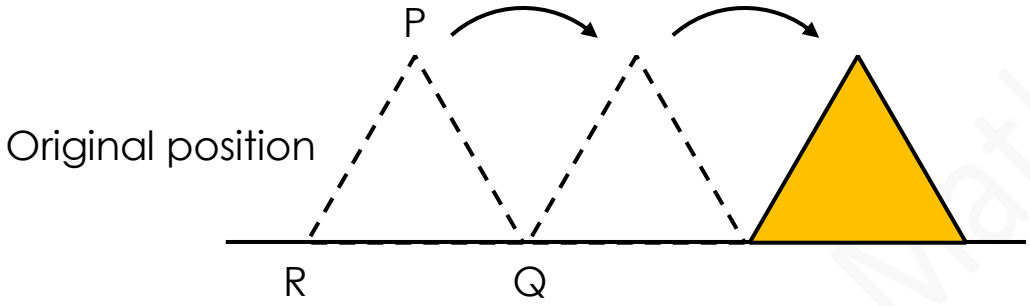
$20 \times 21 = 420$ which is greater than 400.
So, we can try 19×20.
$19 \times 20 = 380$ which is smaller than 400.

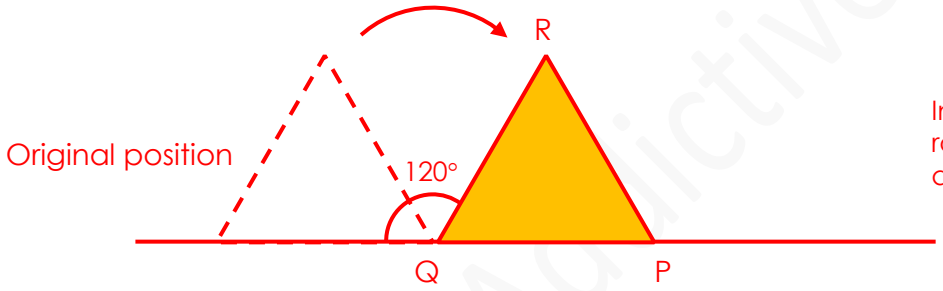
So, the largest possible value of n is 19.

(A) 13 (B) 14 (C) 19
(D) 20 (E) None of the above

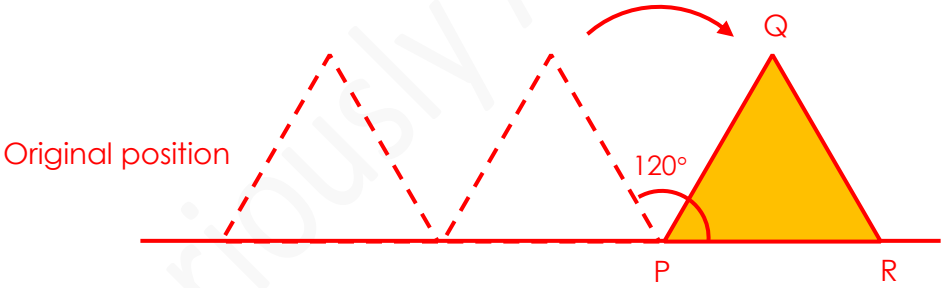
7. Ken placed the base of an equilateral triangular block PQR on flat ground. He rotated the block clockwise about a vertex twice as shown in the diagram. Find the total angle Point R rotated in the clockwise direction.

[Angles in Geometric Shapes / Spatial Visualisation]





In the first rotation, Point R rotated 120° in the clockwise direction.



In the second rotation, Point R rotated 120° in the clockwise direction.

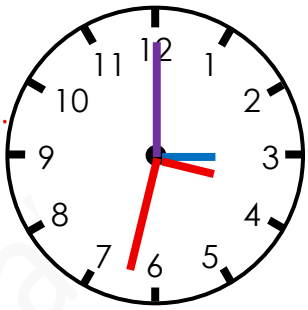
120° + 120° = 240°
Point R rotated 240° in the clockwise direction.

(A) 120° (B) 180° (C) 240°

(D) 360° (E) None of the above

8. The time on the clock shown is 3 o'clock. The minute hand and hour hand are perpendicular to each other. How many minutes will it take for the hands to be perpendicular again?

[Rate / Logical Reasoning]



The minute hand moves 360° in 60 min. It moves $\frac{360°}{60}$ = 6° in 1 min.

The hour hand moves 30° in 60 min. It moves $\frac{30°}{60}$ = 0.5° in 1 min.

The minute hand moves 6° – 0.5° = 5.5° more than the hour hand in 1 min.

For the minute hand to be again perpendicular with the hour hand, it needs to move 180° more than the hour hand (refer to diagram).

$180 \div 5.5 = 180 \div \frac{11}{2}$

$= 180 \times \frac{2}{11}$

$= \frac{360}{11}$

$= 32\frac{8}{11}$

It will take $32\frac{8}{11}$ min for the hands to be again perpendicular.

(A) 30 min (B) $32\frac{1}{2}$ min (C) $32\frac{7}{11}$ min

(D) $32\frac{8}{11}$ min (E) $33\frac{3}{11}$ min

9. A beaker contained a salt solution. 80% of the solution was water. After some time, 25% of the water evaporated. What percentage of the solution in the end was salt?

[Percentage / Restate the Problem]

Let the amount of water be 80 units. There were 20 units of salt.

$\frac{25}{100} \times 80 = 20$

20 units of water evaporated.

80 – 20 = 60

60 units of water remained. 20 units of salt were still in the solution.

$\frac{20}{80} \times 100\% = 25\%$

25% of the solution in the end was salt.

(A) 25% (B) 30% (C) 40%

(D) 60% (E) None of the above

10. There were five teams A, B, C, D and E in a football competition. Each team must play exactly once against another team.

[Logical Reasoning]

a) Team A has played exactly 4 games.

b) Team B has played exactly 3 games.

c) Team C has played exactly 2 games.

d) Team D has played exactly 1 game.

Which of the following statements is **false**?

From **a)**, we know that Team A has played with each team because there are 4 opposing teams and each team only played exactly once against another team.

From **a)** and **d)**, we know that Team D has played 1 game and that was with Team A.

From **a)** and **c)**, we know that Team C has played 2 games and 1 of them was with Team A. The second game was with either Team B or E.

If the second game was with Team E, then Team B must have played with teams A, D and E, which is impossible because Team D only played with Team A.

Therefore, the second game Team C played was with Team B.

Since we know Team B played 3 games, one with Team A and one with Team C, the third game must be with Team E.

So, we have the following:
- Team A has played with teams B, C, D and E.
- Team B has played with teams A, C and E.
- Team C has played with teams A and B.
- Team D has played with Team A.
- Team E has played with teams A and B.

(A) Team A has played with each team exactly once.
(B) Team B has played with Team E.
(C) Team C has played with Team B.
(D) Team D has played with Team A.
(E) Team E has played with Team C.

Section B

Questions 11 and 12 carry 6 marks each.

11. Roy has a scooter with brand new tyres.
The front tyre wears out at a distance of 3600 kilometres.
The back tyre wears out at a distance of 2400 kilometres.
He will stop if either tyre is worn out.
To travel the greatest possible distance, he will switch the tyres to ensure both tyres wear out at the same time.
What is the greatest distance he can travel?

[Average / Simplify the Problem]

$\frac{1}{3600}$ of the front tyre wears out per kilometre travelled.
$\frac{1}{2400}$ of the back tyre wears out per kilometre travelled.

We can find the average rate at which the tyres wear out since he can switch them to ensure both tyres wear out at the same time.

$$\frac{1}{3600} + \frac{1}{2400} = \frac{2}{7200} + \frac{3}{7200}$$
$$= \frac{5}{7200}$$
$$= \frac{1}{1440}$$
$$\frac{1}{1440} \div 2 = \frac{1}{1440} \times \frac{1}{2}$$
$$= \frac{1}{2880}$$

On average, $\frac{1}{2880}$ of each tyre wears out per kilometre travelled.

The greatest distance he can travel is 2880 km.

12. Four pupils, Amy, Ben, Cheryl and Don took a quiz.

There were only 8 questions with answers either True (T) or False (F). Each question is worth 1 mark.

The table below represents their answers and scores.

	Question number								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total marks**
Amy	T	T	F	T	F	T	F	F	5
Ben	F	F	F	T	T	T	F	T	7
Cheryl	T	F	T	F	T	F	T	T	4
Don	F	F	F	T	T	F	T	F	4

Which questions were answered correctly by only two pupils?

[Logical Reasoning]

Observe that when two persons give two different answers to a question, only one of them is correct.
For example, Amy and Ben answered questions 1, 2, 5 and 8 differently, so 4 marks are shared between Amy and Ben.
Ben scored 2 more marks than Amy in total. So, he must have answered 2 more questions correctly than Amy out of the 4 questions they answered differently.
Out of the 4 questions they answered differently, Ben answered 3 questions correctly and Amy answered 1 question correctly.
Ben got 3 marks from these 4 questions and 4 marks from the other 4 questions that he answered the same as Amy.
Now, we know the correct answers for questions 3, 4, 6 and 7.

Cheryl answered questions 3, 4, 6 and 7 wrongly. She scored 4 marks, so, she answered the remaining 4 questions correctly.

The questions answered correctly by only two pupils were 1, 6, 7 and 8.

The answer is 1678.

	Question number								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total marks**
Amy	T	T	F	T	F	T	F	F	5
Ben	F	F	F	T	T	T	F	T	7
Cheryl	T	F	✗	✗	T	✗	✗	T	4
Don	F	F	F	T	T	✗	✗	F	4
Correct answers			F	T		T	F		

*Highlighted cell represents correct answer.
*Cross represents wrong answer.

	Question number								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total marks**
Amy	T	✗	F	T	✗	T	F	✗	5
Ben	✗	F	F	T	T	T	F	T	7
Cheryl	T	F	✗	✗	T	✗	✗	T	4
Don	✗	F	F	T	T	✗	✗	✗	4
Correct answers	T	F	F	T	T	T	F	T	

*Highlighted cell represents correct answer.
*Cross represents wrong answer.